# Optimizing the fundamental limits for quantum communication

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# Quantum capacity of a quantum channel

- The quantum capacity of a channel N is the number of qubits, on average, that can be faithfully transmitted with each use of N.
- The task is to protect quantum information from errors due to quantum noise (or simulate a noiseless quantum channel).


- Quantum capacity theorem is established by (Lloyd, Shor, Devetak 97-05) & (Barnum, Nielsen, Schumacher 96-00)

$$Q(\mathcal{N}) = \lim_{m\to\infty} \frac{1}{m} I_c\left(\mathcal{N}^{\otimes m}\right)$$

- Coherent information $I_c(\mathcal{N}) := \max_\rho \left[ H(\mathcal{N}(\rho)) - H\left(\mathcal{N}^c(\rho)\right)\right]$

# Difficulty of estimating Q(N)

- Q(N) does not have a single-letter formula.
- Regularization is necessary in general [Cubitt el.al, 2014].
- Q(N) is not additive in general [Smith, Yard, 2009].

- Even for qubit depolarizing channel

$$\mathcal{D}_p(\rho) := (1-p)\rho + p\,\mathrm{Tr}(\rho) I / 2$$

we do not know its quantum capacity.

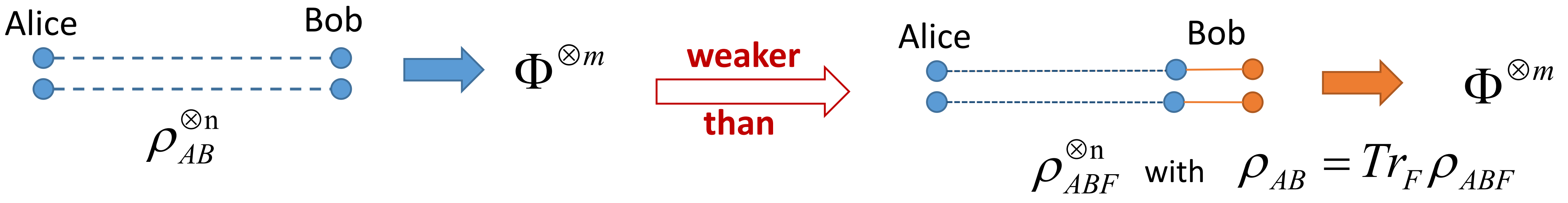
# Many methods to estimate Q(N)

- To evaluate Q(N), substantial efforts have been made in the past two decades.
- Bounds for general channels
  - Partial transposition bound (Holevo, Werner'01)
  - Rains information (Tomamichel, Wilde, Winter'14)
  - max-Rains bound (Wang, Fang, Duan'18)
  - Geometric Rényi bound (Fang, Fawzi'19)
  - ...
- Bounds for depolarizing channel
  - Symmetric Side Channels (Smith, Smolin, Winter'08)
  - Approximate degradable channels (Sutter et al.'17)
  - Degradable decomposition (Leditzky, Datta, Smith'18)
  - Quantum flag bound (Fanizza, Kianvash, Giovannetti'19)

## Main messages

- New single-letter fundamental limits for entanglement distillation, quantum communication, and private communication.
- Optimize the extended channel whose quantum capacity is easy to estimate.
- Improved bounds for several fundamental quantum channels.

# Main result 1 - Bound for distillable entanglement




- Apply the converse bound via approximate degradability bound Leditzky et al. 2017

$$E_{D,\rightarrow}(\rho_{AB}) \leq E_{D,\rightarrow}(\rho_{ABF}) \leq I(A\rangle BF)_\rho + f(\eta(\rho)_{A|BF})$$

where **η (ρ)** is degradability parameter.

- How can we further optimize over the extended states?

Parametrize the extended state!

- Consider the sub-state decomposition $\rho_{AB} = \tau_{AB} + \omega_{AB}$

$$\rho_{ABF} = \tau_{AB} \otimes |\psi_\alpha\rangle\langle\psi_\alpha| + \omega_{AB} \otimes |0\rangle\langle 0| \quad \Rightarrow \quad E_{D,\rightarrow}(\rho_{AB}) \leq \inf_{0\leq\alpha\leq 1} s(\tau_{AB}, \omega_{AB}, \alpha)$$

function s can be efficiently computed.

# Main result 2 - new upper bound for depolarizing channel

- One of the most important channels, useful in modelling noise for quantum hardware.

$$\mathcal{D}_p(\rho) := (1-p)\rho + p\,\mathrm{Tr}(\rho) I/2$$

- However, its quantum capacity remains unsolved despite substantial efforts.

- Q(N) of a teleportation-simulable N = one-way distillable entanglement of its Choi state. (Bennett et al'96)
- The Choi state of the qubit depolarizing channel $\rho_{\mathcal{D}_p} = (1-p)\Phi + pI/4$

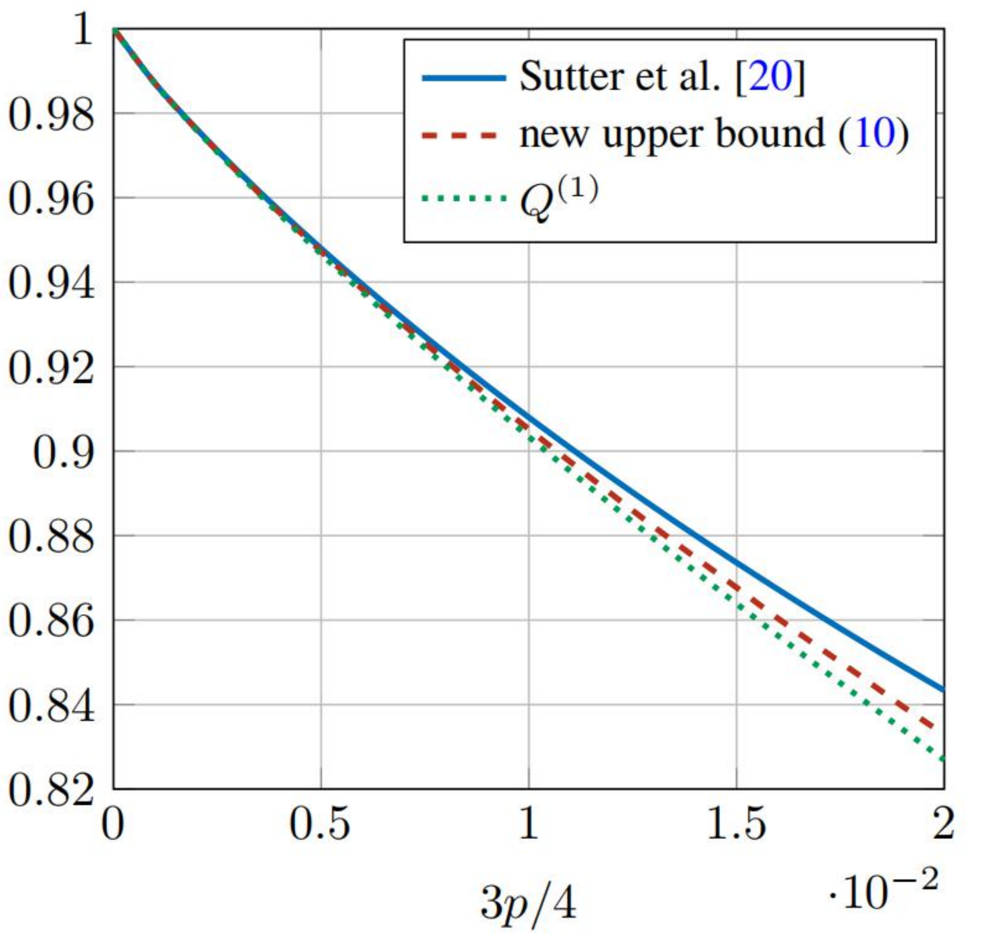
- Applying our bound on one-way distillable entanglement

$$Q\left(\mathcal{D}_p\right) \le \inf_{0\le\alpha\le1} s((1-p)\Phi, pI/4, \alpha)$$

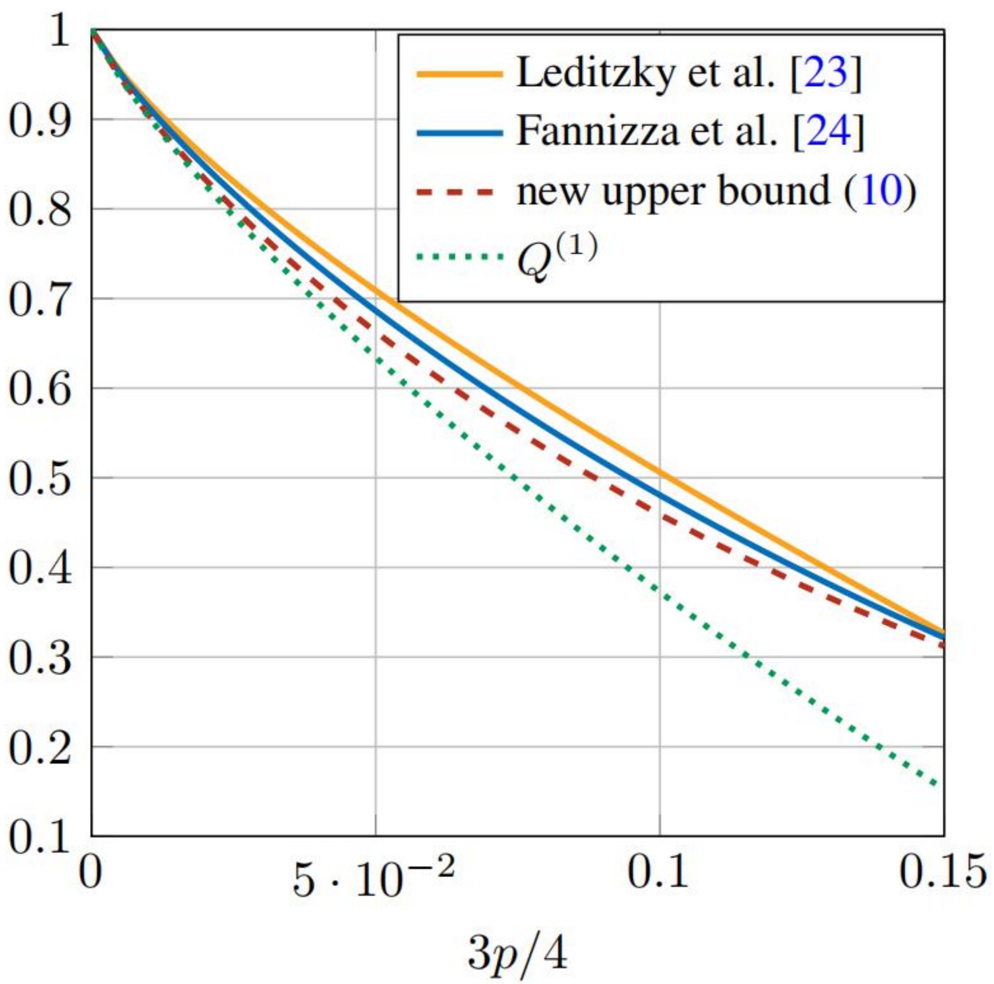
- The final step is to search over α.

# Application 1 - depolarizing channel

- We establish **improved upper bounds** on Q(N) of the depolarizing channel.




Low noise




Intermediate noise

# Upper bounds via flags and degradability

- For a general quantum channel, we could deploy the quantum flags (Fanizza et al.'19).
- For a channel with CP map decomposition, $\mathcal{N} = \sum_{j=0}^{k} \mathcal{N}_j$

$$Q(\mathcal{N}) \leq \inf_{\sigma_0,\cdots,\sigma_k} Q^{(1)}(\widehat{\mathcal{N}}) + f(\eta(\widehat{\mathcal{N}})) \quad \text{with} \quad \widehat{\mathcal{N}}(\cdot) = \sum_{j=0}^{k} \mathcal{N}_j(\cdot) \otimes \sigma_j$$

- We could take a more specific structure $\widehat{\mathcal{N}}(\cdot) = \mathcal{N}_0(\cdot) \otimes |\psi_\alpha\rangle\langle\psi_\alpha| + \mathcal{N}_1(\cdot) \otimes |0\rangle\langle 0|$

$$Q(\mathcal{N}) \leq P(\mathcal{N}) \leq \widetilde{Q}_{pf}(\mathcal{N}) := \inf_{0\leq\alpha\leq 1} \left\{ Q^{(1)}(\widehat{\mathcal{N}}) | \eta(\widehat{\mathcal{N}}) = 0 \right\}$$

- Q1 of degradable extended channels can be efficiently computed (Fawzi & Fawzi'17)

Difference between our method and the quantum flag method in Fanizza et al.'19

1. We parametrize flags and then optimize over them!
2. We consider a general CP map decomposition.

# Application 2 - Generalized admplitude damping channel

- The GAD channel is one of the realistic sources of noise in practice.

$$\mathcal{A}_{y,N}(\rho) = A_1 \rho A_1^\dagger + A_2 \rho A_2^\dagger + A_3 \rho A_3^\dagger + A_4 \rho A_4^\dagger$$

$$A_1 = \sqrt{1-N}(|0\rangle\langle 0| + \sqrt{1-y}\,|1\rangle\langle 1|) \quad A_2 = \sqrt{y(1-N)}\,|0\rangle\langle 1| \quad A_3 = \sqrt{N}(\sqrt{1-y}\,|0\rangle\langle 0| + |1\rangle\langle 1|) \quad A_4 = \sqrt{yN}\,|1\rangle\langle 0|$$

- We introduce the extended channel

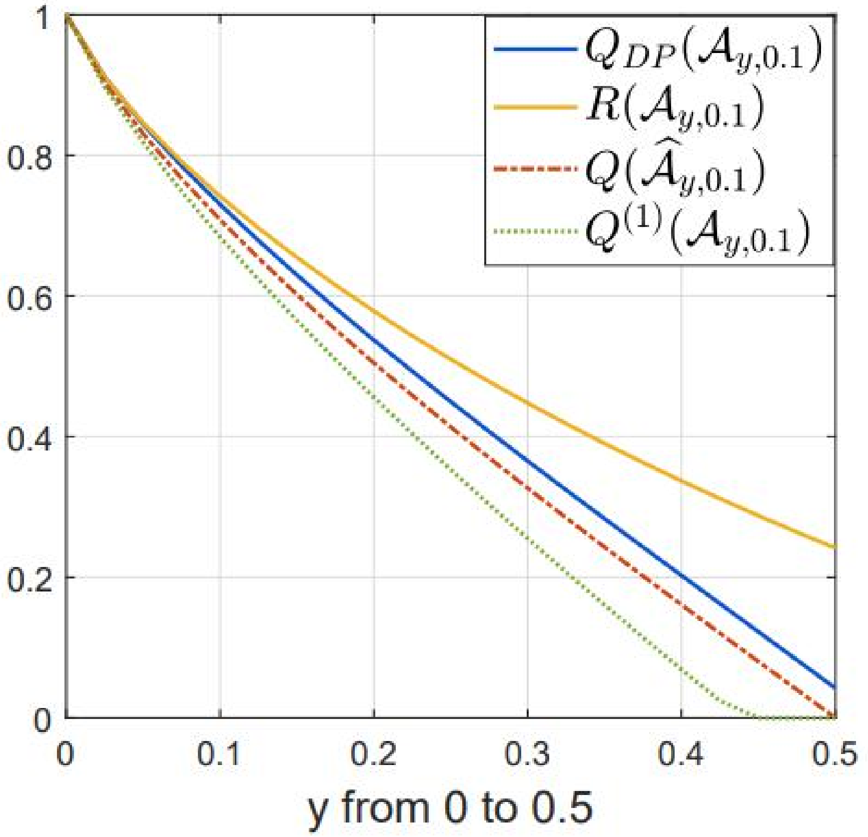
$$\widehat{\mathcal{A}}_{y,N} = \sqrt{1-N}\mathcal{A}^1_{y,N}(\rho) \otimes |\psi_\alpha\rangle\langle\psi_\alpha| + \sqrt{N}\mathcal{A}^2_{y,N}(\rho) \otimes |0\rangle\langle 0|$$

- By numerics and analysis, we find that **α = 0** would be the best choice.
- We further show that

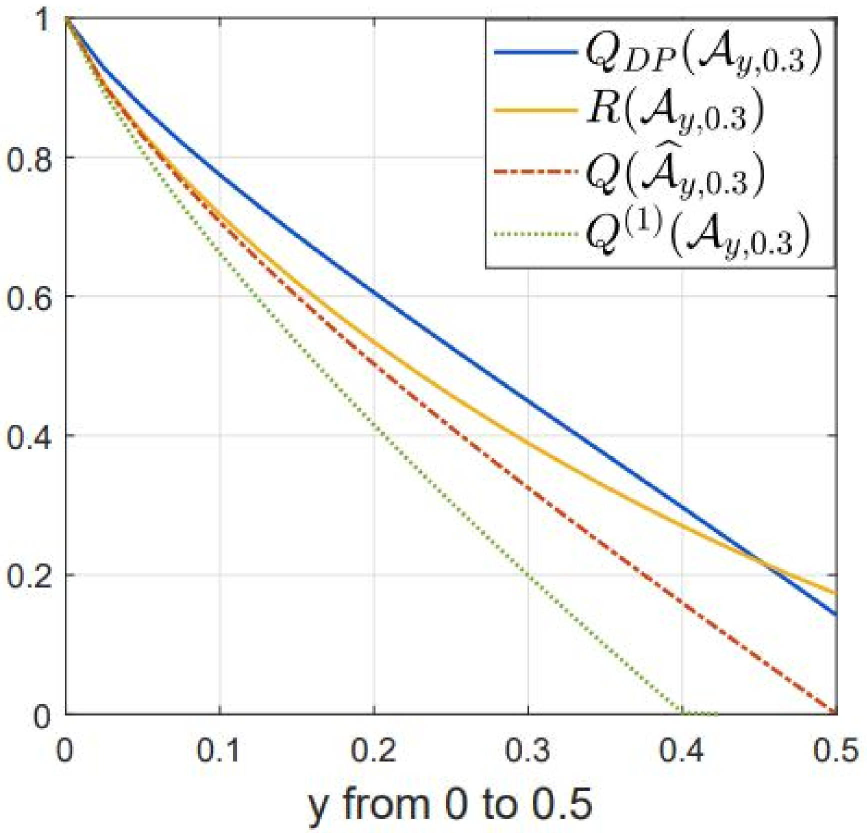
$$Q(\mathcal{A}_{y,N}) \leq Q\left(\widehat{\mathcal{A}}_{y,N}\right) = \max_{p\in[0,1]} I_c\left(p|0\rangle\langle 0| + (1-p)|1\rangle\langle 1|, \widehat{\mathcal{A}}_{\gamma,N}\right)$$

# Application 2 - Generalized admplitude damping channel

- Our bound is tighter than previous upper bounds in (Khatri et al.'19) via the data processing approach (Khatri et al.'19) and Rains information (Tomamichel'14).




(a) $\mathcal{A}_{y,0.1}$

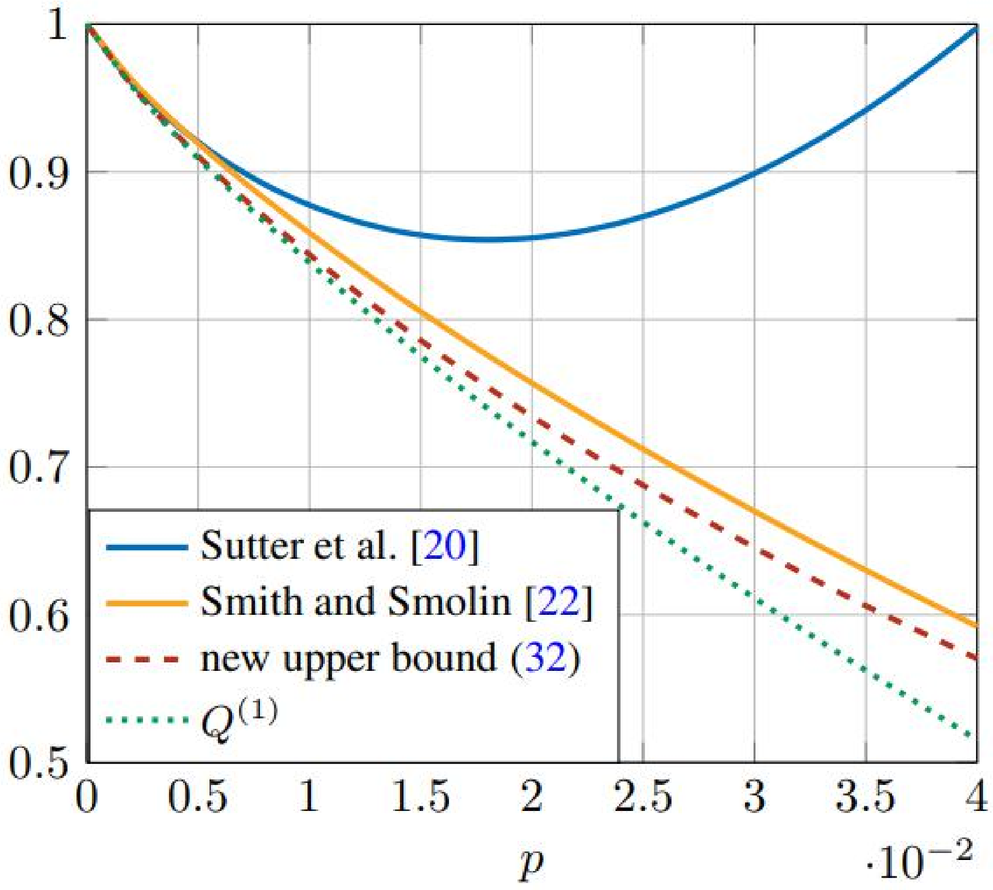



(b) $\mathcal{A}_{y,0.3}$

# Application 3 - BB84 channel

Independent bit and phase error

$$\mathcal{B}_{p_X,p_Z}(\rho) = (1 - p_X - p_Z + p_X p_Z)\rho + (p_X - p_X p_Z)X\rho X + (p_Z - p_Z p_X)Z\rho Z + p_X p_Z Y\rho Y$$




$$Q^{(1)}(\mathcal{B}_{p_X,p_Z}) = 1 - h(p_X)0 - h(p_Z)$$

Smith and Smolin'08

$$Q(\mathcal{B}_{p,p}) \leq h\left(\frac{1}{2} - 2p(1-p)\right) - h(2p(1-p))$$

Sutter et al.'2017 improved the bound in the region 0<p<0.0002

We have established improved bounds for this channel.

# Summary

- Single-letter upper bounds on entanglement distillation + quantum/private communication.
- The key idea is to optimize the extended channels.
- The extended or flagged channel structure is quite useful and can be combined with other techniques of channel capacity estimation.
- Improved upper bounds on the quantum/private capacities of depolarizing channel, BB84 channel, generalized amplitude damping channel.

# Outlook

- It will be interesting to look at the interaction between extended channels and the degradable and anti-degradable decomposition of channels (Leditzky et al.'18).
- Apply the techniques in this work to classical capacity or other resource theories.

# Thanks for your attention!

See arXiv:1912.00931 for more details.

Slides available at www.xinwang.info